Principles to Actions

Ensuring Mathematical Success for All

NCTM®
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Essential Elements

The Mathematics Teaching Practices described and illustrated in the previous section support effective learning for all students. However, although such teaching and learning form the nonnegotiable core of successful mathematics programs, they are part of a system of essential elements of excellent mathematics programs. Consistent implementation of effective teaching and learning of mathematics, as previously described in the eight Mathematics Teaching Practices, are possible only when school mathematics programs have in place—

- a commitment to **access and equity**;
- a powerful **curriculum**;
- appropriate **tools and technology**;
- meaningful and aligned **assessment**; and
- a culture of **professionalism**.

This section describes and illustrates each of these five essential elements of effective school mathematics programs.

Access and Equity

An excellent mathematics program requires that all students have access to a high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential.

> Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students. (NCTM 2000, p. 12)

> Often, inequalities in achievement are perceived as the result of a hierarchy of competence. When the very students who have been given more opportunities to learn show higher achievement than students provided fewer opportunities to learn, they are perceived as more capable or having more aptitude. This manner of talking about achievement gaps without mentioning opportunity gaps that cause them invites a focus on deficit models to "explain" low performance in terms of factors such as cultural differences, poverty, low levels of parental education, and so on. (Flores 2007, p. 40)

Access and equity in mathematics at the school and classroom levels rest on beliefs and practices that empower all students to participate meaningfully in learning mathematics and to achieve outcomes in mathematics that are not predicted by or correlated with student characteristics. These outcomes include performance on mathematics assessments, disposition toward mathematics, persistence in mathematics coursework, and the ability to use mathematics in authentic contexts (Gutiérrez 2002). Support for access and equity requires, but is not limited to, high expectations, access to high-quality mathematics curriculum and instruction, adequate time for students to learn, appropriate emphasis on differentiated processes that broaden students' productive engagement with mathematics, and human and material resources.

Equity in school mathematics outcomes is often conflated with *equality* of inputs. Providing all students the same curricular materials, the same methods of teaching, the same amount of instructional time, and the same school-based supports for learning is different from ensuring that all students, regardless of background characteristics, have the same likelihood of achieving meaningful outcomes (Gutiérrez 2013).

Our vision of access and equity requires being responsive to students' backgrounds, experiences and knowledge when designing, implementing, and assessing the effectiveness of a mathematics program. Acknowledging and addressing factors that contribute to differential outcomes among groups of students is critical to ensure that all students routinely have opportunities to experience high-quality mathematics instruction, learn challenging mathematics content, and receive the support necessary to be successful. Our vision of equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.

Attending to access and equity also means recognizing that mathematics programs that have served some groups of students, in effect privileging some students over others, must be critically examined and enhanced, if needed, to ensure that they meet the needs of *all* students. That is, they must serve students who are black, Latino/a, American Indian, or members of other minorities, as well as those who are considered to be white; students who are female as well as those who are male; students of poverty as well as those of wealth; students who are English language learners as well as those for whom English is their first language; students who have not been successful in school and in mathematics as well as those who have succeeded; and students whose parents have had limited access to educational opportunities as well as those whose parents have had ample educational opportunities. Moreover, attending to access and equity means recognizing that inequitable learning opportunities can exist in any setting, diverse or homogenous, whenever only some, but not all, teachers implement rigorous curricula or use the Mathematics Teaching Practices described earlier.

Abundant research has documented the significant outcomes that are possible when schools and teachers systematically address obstacles to success in mathematics for students from

historically underserved populations (Boaler 1997, 2006; Boaler and Staples 2008; Campbell 1996; Cross et al. 2012; Gutiérrez 2000; Kisker et. al. 2012; Knapp et al. 1995; Lipka et al. 2007; McKenzie et al. 2011). The question is not whether all students can succeed in mathematics but whether the adults organizing mathematics learning opportunities can alter traditional beliefs and practices to promote success for all.

Obstacles

A range of obstacles exists to making significant progress in achieving the Access and Equity Principle. One of these involves the quality of instruction available to students. Researchers have consistently found that students living in poverty, whether urban or rural, as well as students who have struggled to learn mathematics, are more likely to have teachers who have weaker mathematics backgrounds, less professional experience, and certification outside of rather than in mathematics, and who are perceived to be less effective (Battey 2013; Darling-Hammond 2007; Flores 2007; Stiff, Johnson, and Akos 2011). Moreover, in instruction for these students, the Mathematics Teaching Practices described previously are rarely implemented consistently to support meaningful learning. Instead, lessons commonly focus primarily on rote skills and procedures, with scant attention to meaningful mathematics learning (Ellis 2008; Ellis and Berry 2005).

Another obstacle to access and equity involves differential opportunities to learn high-quality grade-level mathematics content and to be held to high expectations for mathematics achievement (Jackson et al. 2013; Phelps et al. 2012; Walker 2003). This often occurs as a result of tracking, or separating students academically on the basis of presumed ability—an unquestioned or commonly tolerated policy that is found in over 85 percent of U.S. schools and limits participation and achievement for students (Biafora and Ansalone 2008). Tracking consigns some students to mathematical content that offers little significant mathematical substance (Burris et al. 2008). While some students are expected to engage in a variety of mathematics topics through multiple teaching and learning strategies, students in low tracks are often confronted with a narrow and fragmented mathematics curriculum, delivered with a limited set of teaching and learning strategies (Ellis 2008; Tate and Rousseau 2002). Too often, because of the unproductive beliefs described below, the capacities of so-called low-track students are underestimated, leading to these students receiving fewer opportunities to learn challenging mathematics. Low-track students encounter a vicious cycle of low expectations: Because little is expected of them, they exert little effort, their halfhearted efforts reinforce low expectations, and the result is low achievement (Gamoran 2011).

Advocates of tracking argue that it assists mathematics teaching and learning by matching students' ability levels to an appropriate curriculum (Schmidt, Cogan, and Houang 2011). The assumption that underlies this belief is that creating different tracks is an effective strategy to accommodate differences in students' needs. The belief is that tracking eases the challenges

of teaching by narrowing the range of student differences so that instructional practices can be targeted to a narrower set of student needs. Implicit in this belief is the idea that students in low-level and high-level tracks would receive few, if any, benefits from being in the same learning environment.

Although some research supports grouping gifted and talented students in homogeneous groups to maximize their learning (Delcourt et al. 1994), research also shows that the learning of students assigned to lower-ability groups is depressed, regardless of their ability levels (Stiff, Johnson, and Akos 2011). In addition, once students are placed in low-level or "slow" math groups, they are very likely to remain in those groups until they leave school (Boaler 2008; Ellis 2008). When middle-level students thought to be "at risk" in mathematics are placed in grade-level mathematics courses and provided the support necessary to be successful in those courses, their achievement gains are greater, and they are more likely to enroll in upper-level math courses in the following years, than when they are placed in lower-ability math courses (Boaler and Staples 2008; Burris, Heubert, and Levin 2006). Further, evidence suggests that high-achieving students in heterogeneous classes are not statistically different from homogeneously tracked students in achievement and participation in Advanced Placement (AP) mathematics courses (Burris, Heubert, and Levin 2006; Staples 2008).

Eliminating low-level tracks does not mean eliminating Advanced Placement or more rigorous high school courses. An effective mathematics program supports and challenges students who have demonstrated strong interest and achievement in mathematics as well as those who have not. However, offering two levels of high school courses, both featuring high-quality curriculum and instruction, is very different from the typical practice of offering multiple levels of the same course (e.g., Algebra 1, Applied Algebra, Algebra 1 Honors, Introductory Algebra, First-Year Fundamental Algebra) with different curricula and expectations (Schmidt, Cogan, and McKnight, 2010). Further, when mathematics programs offer advanced courses, they must ensure that pathways to the highest-level courses exist for all students, along with the support to encourage their participation and success.

Even more disturbing is the lack of self-confidence that far too many students develop and that leads them to view mathematics as something that is far beyond their grasp and that they can never hope to understand. They see mathematics as being within the reach of only a few exceptional "mathematical geniuses." Parents may unwittingly reinforce this notion by excusing low performance by their children as genetic destiny (saying, for example, "I was never any good at math, either"). Furthermore, educators may reinforce this misconception by sorting students by ability, believing that some can "do math" and others cannot.

These obstacles are seldom, if ever, erected purposely to limit the participation or achievement of groups of students. Rather, they emerge in part from a set of beliefs, summarized in the table below, which must be acknowledged and discussed openly. It is important to note that these beliefs should not be viewed as good or bad, but rather as productive when

they lead to change and promote equity or unproductive when they limit student access to important mathematics content and practices. Until unproductive beliefs are confronted, it is unlikely that the goal of mathematical success for all students will be achieved.

The following table compares some unproductive and productive beliefs that influence the access that students have to effective instruction, high-quality curriculum, and differentiated learning supports.

Beliefs about access and equity in mathematics	
Unproductive beliefs	**Productive beliefs**
Students possess different innate levels of ability in mathematics, and these cannot be changed by instruction. Certain groups or individuals have it while others do not.	Mathematics ability is a function of opportunity, experience, and effort—not of innate intelligence. Mathematics teaching and learning cultivate mathematics abilities. All students are capable of participating and achieving in mathematics, and all deserve support to achieve at the highest levels.
Equity is the same as equality. All students need to receive the same learning opportunities so that they can achieve the same academic outcomes.	Equity is attained when students receive the differentiated supports (e.g., time, instruction, curricular materials, programs) necessary to ensure that all students are mathematically successful.
Equity is only an issue for schools with racial and ethnic diversity or significant numbers of low-income students.	Equity—ensuring that all students have access to high-quality curriculum, instruction, and the supports that they need to be successful—applies to all settings.
Students who are not fluent in the English language are less able to learn mathematics and therefore must be in a separate track for English language learners (ELLs).	Students who are not fluent in English can learn the language of mathematics at grade level or beyond at the same time that they are learning English when appropriate instructional strategies are used.
Mathematics learning is independent of students' culture, conditions, and language, and teachers do not need to consider any of these factors to be effective.	Effective mathematics instruction leverages students' culture, conditions, and language to support and enhance mathematics learning.
Students living in poverty lack the cognitive, emotional, and behavioral characteristics to participate and achieve in mathematics.	Effective teaching practices (e.g., engaging students with challenging tasks, discourse, and open-ended problem solving) have the potential to open up greater opportunities for higher-order thinking and for raising the mathematics achievement of all students, including poor and low-income students.

Beliefs about access and equity in mathematics, *continued*	
Unproductive beliefs	**Productive beliefs**
Tracking promotes students' achievement by allowing students to be placed in "homogeneous" classes and groups where they can make the greatest learning gains.	The practice of isolating low-achieving students in low-level or slower-paced mathematics groups should be eliminated.
Only high-achieving or gifted students can reason about, make sense of, and persevere in solving challenging mathematics problems.	All students are capable of making sense of and persevering in solving challenging mathematics problems and should be expected to do so. Many more students, regardless of gender, ethnicity, and socioeconomic status, need to be given the support, confidence, and opportunities to reach much higher levels of mathematical success and interest.

Overcoming the obstacles

Achieving equity with respect to student learning outcomes will require that educators at all levels operate with the belief that all students can learn. Closing existing learning gaps requires ensuring that all students have access to high-quality instruction, a challenging curriculum, exciting extracurricular opportunities, and the differentiated supports and enrichment that are necessary to promote student success at continually increasing levels.

Beliefs and expectations

To ensure that all students have access to an equitable mathematics program, educators need to identify, acknowledge, and discuss the mindsets and beliefs that they have about students' abilities. Fixed mindsets (i.e., the attitude that levels of mathematics ability are fixed and cannot be changed), when coupled with societal stereotypes about academic ability that are based on student characteristics, perpetuate the unproductive practices described above (Dweck 2008). In contrast, a growth mindset, which emphasizes mathematics teaching and learning as processes that cultivate mathematical abilities, stresses that success and learning are a reflection of effort and not intelligence alone, and thus promotes a belief that all students are capable of participating and achieving in mathematics (Boaler 2011; Dweck 2006).

Believing in, and acting on, growth mindsets versus fixed mindsets can make an enormous difference in what students accomplish. Setting and acting on high expectations and a genuine belief that student effort and effective instruction outweigh "smarts" and circumstances increase students' opportunities to learn. Teachers with fixed mindsets can unfairly justify differential allocation of resources and opportunities on the basis of students'

prior academic achievement, abilities, or interests. Research has found that a fixed mindset is strongly correlated with socioeconomic background, contributes to widening opportunity gaps, and reinforces inequities (Dweck 2008; Gamoran 2010). To address this obstacle, teachers should promote and display a growth mindset at all times. A growth mindset values all students' thinking and uses pedagogical practices such as differentiated tasks, mixed-ability groupings, and public praise for contributions and perseverance to cultivate mathematical participation and achievement (Boaler 2011).

Promoting student engagement (by, e.g., selecting challenging tasks, exerting intense effort and concentration in the implementation of tasks), framing mathematics within the growth mindset, acknowledging student contributions, and attending to culture and language play substantial roles in equalizing mathematics gains between poor and non-poor students (Battey 2013; Cross et al. 2012; Kisker et al. 2012; Robinson 2013). Furthermore, increasing access of poor and low-income students to teaching that effectively enacts the Mathematics Teaching Practices described earlier has the potential to open up greater opportunities for higher-order thinking and for raising the intellectual quality of student cognition (Boaler and Staples 2008; Burris et al. 2008; Lubienski 2007).

With a systemic commitment to all students and expectations that all students can meet or exceed grade-level standards for mathematics, educators can more easily move away from past practices, such as tracking that separated students, and instead develop productive practices that support learning for all.

Curriculum and instruction

When differences in ability, background, and interest arise, as they always will, more effective instruction and differentiated supports can overcome the obstacles discussed above. Policies that boost and supplement learning, provide additional time, and give students access to a rigorous curriculum and teachers who implement a range of approaches and resources are far more likely to raise achievement than policies that relegate students who have traditionally underperformed to dead-end tracks with an unchallenging curriculum.

Persistent and unacceptable gaps narrow and ultimately disappear when all students have access to rigorous, high-quality mathematics, taught by teachers who not only understand mathematics but also understand and appreciate learners' social and cultural contexts in meaningful ways. Effective teachers draw on community resources to understand how they can use contexts, culture, conditions, and language to support mathematics teaching and learning (Berry and Ellis 2013; Cross et al. 2012; Kisker et al. 2012; Moschkovich 1999, 2011; Planas and Civil 2013). As a result, learning mathematics becomes a part of a student's sense of identity, leading to increased engagement and motivation in mathematics (Aguirre, Mayfield-Ingram, and Martin 2013; Boaler 1997; Hogan 2008; Middleton and Jansen 2011).

Classroom environments that foster a sense of community that allows students to express their mathematical ideas—together with norms that expect students to communicate their mathematical thinking to their peers and teacher, both orally and in writing, using the language of mathematics—positively affect participation and engagement among all students (Horn 2012; Webel 2010). All students, including ELLs, can learn mathematics content at the same time that they are learning the academic language of mathematics, both in English and in symbols (Razfar, Khisty, and Chval 2011). The language of mathematics provides an opportunity for many students, including ELLs, to show their prior preparation and to help one another in the language that they have in common—the language of mathematics (Moschkovich 1999, 2011).

Furthermore, a focus on the mathematical practices outlined in CCSSM can benefit students at all levels by engaging them in doing mathematics in ways that make sense to them. Rather than imposing a standard algorithm or a set solution strategy, students can devise their own strategies that are more meaningful to them, easier to remember, or culturally familiar (Carpenter et al. 1989). Particularly useful in this endeavor are problems that have multiple entry points and allow for the use of a broad range of strategies or approaches. More advanced students can extend their thinking as they work with problems with multiple entry points, while less advanced students, including students with disabilities, have opportunities to continue to develop basic understandings that they need to move forward (Dieker et al. 2011). Moreover, problems that students can enter and reason about at multiple levels can accommodate a range of learning styles and cultural backgrounds.

Interventions and support personnel

Supporting the success of all students requires having an effective intervention program in place to address learning difficulties as soon as they occur. Although specific program design features will vary by level and other factors, effective intervention programs should—

- be mandatory, not optional (i.e., scheduled during the school day whenever possible);
- be based on constant monitoring of students' progress, as determined from the results of formative and summative assessment, ensuring that students get support as quickly as possible;
- attend to conceptual understanding as well as procedural fluency; and
- allow for flexible movement in and out of the intervention as students need it (Kanold and Larson 2012).

One option is to provide such intervention during regular mathematics instructional time. For example, elementary teachers of the same grade may decide to schedule their ninety-minute math block at the same time so that they can use the first twenty minutes of each period for mathematics intervention—with students regrouped across classes according to their learning needs and then returning to their heterogeneous class for the regular mathematics lesson.

Another option is to allocate additional time outside the grade-level mathematics course during which students with learning gaps can receive specific, targeted support (Burris et al. 2008; Rubin and Noguera 2004). These additional learning opportunities should enable these students to explore math on a deep, intriguing, innovative level. These sessions might be offered during a "double-dose" math time as well as outside the regular school day. In addition to regular curricular support, engaging co-curricular and extracurricular opportunities, such as mathematics clubs, circles, and competitions, as well as access to mentors, can help students achieve the highest levels of mathematical passion, creativity, and expertise, regardless of gender, ethnicity, or socioeconomic status. These resources should enable students not only to see beyond math simply as a school subject, but also to appreciate the beauty, wonder, utility, and vitality of mathematics at a deep level, helping them to incorporate it into their future high-level decision making.

Another strategy for promoting equitable, full access to opportunities to learn mathematics is the deployment of instructional support personnel (for example, mathematics resource teachers, intervention teachers, or gifted specialists) who can provide specialized support services to schools and teachers or can work directly with students who are either underperforming or exceeding grade-level standards of proficiency or who display curiosity and desire for learning additional mathematics. Schools serving learners in diverse contexts with diverse learning needs can use the assistance of school-based mathematics coaches and specialists to enhance teachers' abilities and capacities to meet individual students' learning needs, improve instruction, and monitor students' progress. Mathematics coaches and specialists can positively influence teachers' beliefs about mathematics teaching and learning and increase teachers' participation in non-coaching professional activities, such as attending mathematics-focused grade-level meetings, observing peers' teaching, or attending schoolwide mathematics workshops (Campbell and Malkus 2011).

Illustration

The following example illustrates an intervention at the high school level to ensure that all students continue to move forward, learning challenging, high-level mathematics:

> Teachers and administrators at a high school in a mid-Atlantic state became aware that a significant number of ninth graders were not succeeding in Algebra 1. To deal with this concern, the mathematics teachers and the school administrators began meeting regularly to brainstorm about ways to address this inequity. In reviewing the records of these students, they found that the correlation between the students' achievement on the eighth-grade state assessment and their performance in Algebra 1 in ninth grade was extremely high. The teachers concluded that many of these students were likely to have gaps in their knowledge that prevented them from achieving to their fullest potential when they took Algebra 1.

To address this problem, the school designed a new course, Algebra Seminar, for approximately 20 percent of its ninth-grade students—primarily those scoring at a "basic" or "below basic" level on the eighth-grade assessment and therefore deemed unlikely to pass the Algebra 1 end-of-course exam if they enrolled in a typical one-period Algebra 1 class. To ensure that the students in this course would receive the appropriate levels of support, the principal agreed to schedule common planning time for Algebra Seminar teachers so that they could collaboratively design the course, plan lessons, and enhance pedagogical practices.

Multiple design features of the new Algebra Seminar course make it an effective intervention that meets the vision of the Access and Equity Principle. The new course—

- teaches Algebra 1 course content along with critical prerequisite content, with a "just-in-time" approach to prerequisite content;
- is team-taught by a mathematics teacher and a special education teacher to ensure that the special needs students who are mainstreamed into the class receive the additional support that they need to succeed;
- is systematically planned as a back-to-back double period (ninety minutes a day);
- is capped at eighteen students, so that teachers have the opportunity to address individual students' needs;
- is enriched by focused professional development for the teachers;
- uses a broad array of print and non-print, and basal and supplemental, resources;
- engages students and enhances instruction with a variety of tools and technology, including interactive whiteboards, graphing calculators, tablet computers, response clickers, and a range of manipulative materials;
- incorporates a wide variety of highly effective instructional practices that reflect the Mathematics Teaching Practices; and
- draws on online lesson plans and other resources that teachers use to initiate their planning.

As a result of this comprehensive and well-designed intervention, Algebra Seminar students consistently catch up and perform as well as the single-period Algebra 1 students on the end-of-course Algebra 1 exam and are prepared to enroll in a regular geometry course the following year.

Moving to action

To provide access and equity, teachers go beyond "good teaching," to teaching that ensures that all students have opportunities to engage successfully in the mathematics classroom and

learn challenging mathematics. Making this Principle a reality requires all stakeholders to monitor the extent to which all students have access to a challenging mathematics curriculum, taught by skilled and effective teachers who know and understand the cultures and communities from which their students come and who also use this knowledge to create meaningful tasks that build on students' prior knowledge and experiences. These teachers also monitor student progress and make needed accommodations. To do this effectively, they work collaboratively with colleagues, including teachers of special education, gifted education, and English language learners, as well as families and community members, to ensure that all students have the support that they need to maximize their success in the mathematics classroom. Further, teachers need to collaborate with one another to implement the Mathematics Teaching Practices outlined earlier and promote a growth mindset in their classrooms and school.

Finally, district and school policies must be reviewed to ensure that systemic practices are not disadvantaging particular groups or subgroups of students on the basis of societal stereotypes. This analysis should include a review of tracking, student placement, opportunities for both remediation and enrichment, and student outcomes, including persistence in the mathematics pipeline.