Principles to Actions

Ensuring Mathematical Success for All

NCTM
National Council of Teachers of Mathematics

Progress and Challenge

In 1989, the National Council of Teachers of Mathematics (NCTM) launched the standards-based education movement in North America with the release of *Curriculum and Evaluation Standards for School Mathematics*, an unprecedented initiative to promote systemic improvement in mathematics education. Now, twenty-five years later, the widespread adoption of college- and career-readiness standards, including adoption in the United States of the Common Core State Standards for Mathematics (CCSSM) by forty-five of the fifty states, provides an opportunity to reenergize and focus our commitment to significant improvement in mathematics education. To realize the potential of these new standards, we must examine the progress that has already been made, the challenges that remain, and the actions needed to truly ensure mathematical success for all students.

Looking back at mathematics education and student achievement in mathematics, we find much to celebrate. Owing in large measure to the leadership of NCTM, the gradual implementation of a growing body of research on teaching and learning mathematics, and the dedicated efforts of nearly two million teachers of mathematics in North America, student achievement is at historic highs:

- The percentage of fourth graders scoring "proficient" or above on the National Assessment of Educational Progress (NAEP) rose from 13 percent in 1990 to 42 percent in 2013. (National Center for Education Statistics [NCES] 2013)
- The percentage of eighth graders scoring "proficient" or above on the NAEP rose from 15 percent in 1990 to 36 percent in 2013. (NCES 2013)
- Average scores for fourth and eighth graders on these NAEP assessments rose 29 and 22 points, respectively, between 1990 and 2013. (NCES 2013)
- Between 1990 and 2013, the mean SAT-Math score increased from 501 to 514, and the mean ACT-Math score increased from 19.9 to 20.9. (College Board 2013a; ACT 2013)
- The number of students taking Advanced Placement Calculus examinations increased from 77,634 in 1982 to 387,297 in 2013, of whom about 50 percent scored 4 or 5. (College Board 2013b)
- The number of students taking the Advanced Placement Statistics examination increased from 7,667 in 1997 to 169,508 in 2013, of whom over 33 percent scored 4 or 5. (College Board 2013b)

These are impressive accomplishments. However, while we celebrate these record high NAEP scores and increases in SAT and ACT achievement—despite a significantly larger and more diverse range of test-takers—other recent data make it clear that we are far from where we need to be and that much still remains to be accomplished:

- Average mathematics NAEP scores for 17-year-olds have been essentially flat since 1973. (NCES 2009)
- The difference in average NAEP mathematics scores between white and black and white and Hispanic 9- and 13-year-olds has narrowed somewhat between 1973 and 2012 but remains between 17 and 28 points. (NCES 2013)
- Only about 44 percent of U.S. high school graduates in 2013 were considered ready for college work in mathematics, as measured by ACT and SAT scores. (ACT 2013; College Board 2013c)
- Among cohorts of 15-year-olds from the 34 countries participating in the 2012 Programme for International Student Assessment (PISA), which measures students' capacity to formulate, employ, and interpret mathematics in a variety of real-world contexts, the Canadian cohort ranked 13th in mathematics, placing it quite high among non–East Asian countries, whereas the U.S. cohort ranked 26th. (Organisation for Economic Co-operation and Development [OECD] 2013a)
- Although many countries' mean scores on the PISA assessments increased from 2003 to 2012, the United States' and Canada's mean scores declined. (OECD 2013a)
- U.S. students performed relatively well on PISA items that required only lower-level skills—reading and simple handling of data directly from tables and diagrams, handling easily manageable formulas—but they struggled with tasks involving creating, using, and interpreting models of real-world situations and using mathematical reasoning. (OECD 2013b)
- On the PISA tests, only 8.8 percent of students in the United States reached the top two mathematics levels, compared with 12.6 percent of the students across all 34 participating countries, including 16.4 percent of students in Canada and more than 30 percent of students in Hong Kong–China, Korea, Singapore, and Chinese Taipei. (OECD, 2013a)
- Only 16 percent of U.S. high school seniors are proficient in mathematics and interested in a STEM career. (U.S. Department of Education 2014).

These more disturbing data point to the persistent challenges and the work that we still need to do to make mathematics achievement a reality for all students:

- Eliminate persistent racial, ethnic, and income achievement gaps so that all students have opportunities and supports to achieve high levels of mathematics learning

- Increase the level of mathematics learning of all students, so that they are college and career ready when they graduate from high school
- Increase the number of high school graduates, especially those from traditionally underrepresented groups, who are interested in, and prepared for, STEM careers

In short, we must move from "pockets of excellence" to "systemic excellence" by providing mathematics education that supports the learning of all students at the highest possible level.

To achieve this goal, we must change a range of troubling and unproductive realities that exist in too many classrooms, schools, and districts. *Principles to Actions* discusses and documents these realities:

- Too much focus is on learning procedures without any connection to meaning, understanding, or the applications that require these procedures.
- Too many students are limited by the lower expectations and narrower curricula of remedial tracks from which few ever emerge.
- Too many teachers have limited access to the instructional materials, tools, and technology that they need.
- Too much weight is placed on results from assessments—particularly large-scale, high-stakes assessments—that emphasize skills and fact recall and fail to give sufficient attention to problem solving and reasoning.
- Too many teachers of mathematics remain professionally isolated, without the benefits of collaborative structures and coaching, and with inadequate opportunities for professional development related to mathematics teaching and learning.

As a result, too few students—especially those from traditionally underrepresented groups—are attaining high levels of mathematics learning.

Thus, this is no time to rest on laurels. Even a casual review of entry-level workplace expectations and the daily responsibilities of household management and citizenship suggest that such core mathematical ideas as proportion, rate of change, equality, dimension, random sample, and correlation must be understood by nearly all adults—a target far from the current reality.

What is different and promising today, however, is the hope that the implementation of CCSSM, and the new generation of aligned and rigorous assessments, will help to address the continuing challenges and expand the progress already made. The need for coherent standards that promote college and career readiness has been endorsed across all states and provinces, whether or not they have adopted CCSSM. As NCTM (2013) has publicly declared,

> The widespread adoption of the Common Core State Standards for Mathematics presents an unprecedented opportunity for systemic improvement in mathematics education in the United States. The Common Core State Standards offer a foundation for the development of more rigorous, focused, and coherent mathematics curricula, instruction, and assessments that promote conceptual understanding and reasoning as well as skill fluency. This foundation will help to ensure that all students are ready for college and the workplace when they graduate from high school and that they are prepared to take their place as productive, full participants in society.

CCSSM provides guidance and direction, and helps focus and clarify common outcomes. It motivates the development of new instructional resources and assessments. But CCSSM does not tell teachers, coaches, administrators, parents, or policymakers what to do at the classroom, school, or district level or how to begin making essential changes to implement these standards. Moreover, it does not describe or prescribe the essential conditions required to ensure mathematical success for all students. Thus, the primary purpose of *Principles to Actions* is to fill this gap between the development and adoption of CCSSM and other standards and the enactment of practices, policies, programs, and actions required for their widespread and successful implementation. Its overarching message is that effective teaching is the nonnegotiable core that ensures that all students learn mathematics at high levels and that such teaching requires a range of actions at the state or provincial, district, school, and classroom levels.

In *Principles to Actions*, NCTM sets forth a set of strongly recommended, research-informed actions for all teachers, coaches, and specialists in mathematics; all school and district administrators; and all educational leaders and policymakers. These recommendations are based on the Council's core principles. In *Principles and Standards for School Mathematics*, NCTM (2000) first defined a set of Principles that "describe features of high-quality mathematics education" (p. 11). The list on the following page presents updated Principles that constitute the foundation of *Principles to Actions*.

The revisions to this updated set of Principles reflect more than a decade of experience and new research evidence about excellent mathematics programs, as well as significant obstacles and unproductive beliefs that continue to compromise progress. In succeeding sections, these six Principles are defined, examined for unproductive and productive beliefs, linked to effective practices, and illuminated with examples. The final section proposes specific actions for productive practices and policies that are essential for widespread implementation of pre-K–12 mathematics programs with the power to ensure mathematical success for all students at last.

Guiding Principles for School Mathematics
Teaching and Learning. An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.
Access and Equity. An excellent mathematics program requires that all students have access to a high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential.
Curriculum. An excellent mathematics program includes a curriculum that develops important mathematics along coherent learning progressions and develops connections among areas of mathematical study and between mathematics and the real world.
Tools and Technology. An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking.
Assessment. An excellent mathematics program ensures that assessment is an integral part of instruction, provides evidence of proficiency with important mathematics content and practices, includes a variety of strategies and data sources, and informs feedback to students, instructional decisions, and program improvement.
Professionalism. In an excellent mathematics program, educators hold themselves and their colleagues accountable for the mathematical success of every student and for their personal and collective professional growth toward effective teaching and learning of mathematics.

Effective Teaching and Learning

An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.

The teaching of mathematics is complex. It requires teachers to have a deep understanding of the mathematical knowledge that they are expected to teach (Ball, Thames, and Phelps 2008) and a clear view of how student learning of that mathematics develops and progresses across grades (Daro, Mosher, and Corcoran 2011; Sztajn et al. 2012). It also requires teachers to be skilled at teaching in ways that are effective in developing mathematics learning for all students. This section presents, describes, and illustrates a set of eight research-informed teaching practices that support the mathematics learning of all students. Before turning to these teaching practices, however, we must be clear about the mathematics learning such teaching must inspire and develop and the inextricable connection between teaching and learning.

The learning of mathematics has been defined to include the development of five interrelated strands that, together, constitute mathematical proficiency (National Research Council 2001):

1. Conceptual understanding
2. Procedural fluency
3. Strategic competence
4. Adaptive reasoning
5. Productive disposition

Conceptual understanding (i.e., the comprehension and connection of concepts, operations, and relations) establishes the foundation, and is necessary, for developing procedural fluency (i.e., the meaningful and flexible use of procedures to solve problems).

Strategic competence (i.e., the ability to formulate, represent, and solve mathematical problems) and adaptive reasoning (i.e., the capacity to think logically and to justify one's thinking) reflect the need for students to develop mathematical ways of thinking as a basis for solving mathematics problems that they may encounter in real life, as well as within mathematics and other disciplines. These ways of thinking are variously described as "processes" (in NCTM's [2000] Process Standards), "reasoning habits" (NCTM 2009), or "mathematical practices" (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA Center and CCSSO] 2010). In this publication, in alignment with the Common

Core State Standards for Mathematics (CCSSM), we refer to them as "mathematical practices," which represent what students are doing as they learn mathematics (see fig. 1).

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Fig. 1. Standards for Mathematical Practice (NGO Center and CCSSO 2010, pp. 6–8)

The fifth strand identified on the preceding page, productive disposition, is "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (National Research Council 2001, p. 131). Students need to recognize the value of studying mathematics and believe that they are capable of learning mathematics through resolve and effort (Schunk and Richardson 2011). This conviction increases students' motivation and willingness to persevere in solving challenging problems in the short term and continuing their study of mathematics in the long term. Interest and curiosity evoked throughout the study of mathematics can spark a lifetime of positive attitudes toward the subject.

Student learning of mathematics "depends fundamentally on what happens inside the classroom as teachers and learners interact over the curriculum" (Ball and Forzani 2011, p. 17). Ball and other researchers (e.g., Ball et al. 2009; Grossman, Hammerness, and McDonald 2009; Lampert 2010; McDonald, Kazemi, and Kavanagh 2013) argue that the profession of teaching needs to identify and work together toward the implementation of a common set of high-leverage practices that underlie effective teaching. By "high-leverage practices," they mean "those practices at the heart of the work of teaching that are most likely to affect student learning" (Ball and Forzani 2010, p. 45).

Although effective teaching of mathematics may have similarities with productive teaching in other disciplines (Duit and Treagust 2003; Hlas and Hlas 2012), each discipline requires focused attention on those teaching practices that are most effective in supporting student learning specific to the discipline (Hill et al. 2008; Hill, Rowan, and Ball 2005). Research from both cognitive science (Mayer 2002; Bransford, Brown, and Cocking 2000; National

Research Council 2012a) and mathematics education (Donovan and Bransford 2005; Lester 2007) supports the characterization of mathematics learning as an active process, in which each student builds his or her own mathematical knowledge from personal experiences, coupled with feedback from peers, teachers and other adults, and themselves. This research has identified a number of principles of learning that provide the foundation for effective mathematics teaching. Specifically, learners should have experiences that enable them to—

- engage with challenging tasks that involve active meaning making and support meaningful learning;
- connect new learning with prior knowledge and informal reasoning and, in the process, address preconceptions and misconceptions;
- acquire conceptual knowledge as well as procedural knowledge, so that they can meaningfully organize their knowledge, acquire new knowledge, and transfer and apply knowledge to new situations;
- construct knowledge socially, through discourse, activity, and interaction related to meaningful problems;
- receive descriptive and timely feedback so that they can reflect on and revise their work, thinking, and understandings; and
- develop metacognitive awareness of themselves as learners, thinkers, and problem solvers, and learn to monitor their learning and performance.

Mathematics Teaching Practices

Eight Mathematics Teaching Practices provide a framework for strengthening the teaching and learning of mathematics. This research-informed framework of teaching and learning reflects the learning principles listed above, as well as other knowledge of mathematics teaching that has accumulated over the last two decades. The list on the following page identifies these eight Mathematics Teaching Practices, which represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics.

Obstacles

Dominant cultural beliefs about the teaching and learning of mathematics continue to be obstacles to consistent implementation of effective teaching and learning in mathematics classrooms (Handal 2003; Philipp 2007). Many parents and educators believe that students should be taught as they were taught, through memorizing facts, formulas, and procedures and then practicing skills over and over again (e.g., Sam and Ernest 2000). This view perpetuates the traditional lesson paradigm that features review, demonstration, and practice and is still pervasive in many classrooms (Banilower et al. 2006; Weiss and Pasley 2004). Teachers, as well

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

as parents, are often not convinced that straying from these established beliefs and practi
will be more effective for student learning (Barkatsas and Malone 2005; Wilken 2008).

In sharp contrast to this view is the belief that mathematics lessons should be centered on engaging students in solving and discussing tasks that promote reasoning and problem sol (NCTM 2009; National Research Council 2012a). Teachers who hold this belief plan lessons to prompt student interactions and discourse, with the goal of helping students make sense of mathematical concepts and procedures. However, the lack of agreement about what consti effective mathematics teaching constrains schools and school systems from establishing co ent expectations for high-quality, productive teaching of mathematics (Ball and Forzani 20

Teachers' beliefs influence the decisions that they make about the manner in which they teach mathematics, as indicated in the table at the right. Students' beliefs influence the

Beliefs about teaching and learning mathematics	
Unproductive beliefs	**Productive beliefs**
Mathematics learning should focus on practicing procedures and memorizing basic number combinations.	Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.
Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.	All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.
Students can learn to apply mathematics only after they have mastered the basic skills.	Students can learn mathematics through exploring and solving contextual and mathematical problems.
The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.	The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.
The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.	The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.
An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.	An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

perception of what it means to learn mathematics and their dispositions toward the subject. As the table summarizes, the impact of these beliefs on the teaching and learning of mathematics may be unproductive or productive. It is important to note that these beliefs should not be viewed as good or bad. Instead, beliefs should be understood as unproductive when they hinder the implementation of effective instructional practice or limit student access to important mathematics content and practices.

Overcoming the obstacles

Teaching mathematics requires specialized expertise and professional knowledge that includes not only knowing mathematics but knowing it in ways that make it useful for the work of teaching (Ball and Forzani 2010; Ball, Thames, and Phelps 2008). Mathematics teaching demands subject-specific understanding and insight so that teachers can skillfully carry out their work in mathematics classrooms. Some of the work of mathematics teaching includes finding an example or task to make a specific mathematical point, linking mathematical representations to underlying ideas and other representations, and evaluating students' mathematical reasoning and explanations. This work also requires teachers to be able to unpack mathematical topics that they know well and to reexamine these through the eyes of learners as well as to be able to work with many learners simultaneously in classrooms, each with unique backgrounds, interests, and learning needs.

The following discussion and illustrations of the eight Mathematics Teaching Practices support the incorporation of the productive beliefs identified above into the daily professional work of effective teachers of mathematics. This framework offers educators within schools and across districts a common lens for collectively moving toward improved instructional practice and for supporting one another in becoming skilled at teaching in ways that matter for ensuring successful mathematics learning for all students.

Establish Mathematics Goals to Focus Learning

Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

Effective mathematics teaching begins with a shared understanding among teachers of the mathematics that students are learning and how this mathematics develops along learning progressions. This shared understanding includes clarifying the broader mathematical goals that guide planning on a unit-by-unit basis, as well as the more targeted mathematics goals that guide instructional decisions on a lesson-by-lesson basis. The establishment of clear goals not only guides teachers' decision making during a lesson but also focuses students' attention on monitoring their own progress toward the intended learning outcomes.

Discussion

Mathematics goals indicate what mathematics students are to learn and understand as a result of instruction (Wiliam 2011). In fact, "formulating clear, explicit learning goals sets the stage for everything else" (Hiebert et al. 2007, p. 57). Goals should describe what mathematical concepts, ideas, or methods students will understand more deeply as a result of instruction and identify the mathematical practices that students are learning to use more proficiently. Teachers need to be clear about how the learning goals relate to and build toward rigorous standards, such as the Common Core State Standards for Mathematics. The goals that guide instruction, however, should not be just a reiteration of a standard statement or cluster but should be more specifically linked to the current classroom curriculum and student learning needs, referring, for example, to particular visual representations or mathematical concepts and methods that students will come to understand as a result of instruction.

Learning goals situated within mathematics learning progressions (Daro, Mosher, and Corcoran 2011) and connected to the "big ideas" of mathematics (Charles 2005) provide a stronger basis for teachers' instructional decisions. Learning progressions or trajectories describe how students make transitions from their prior knowledge to more sophisticated understandings. The progressions also identify intermediate understandings and link research on student learning to instruction (Clements and Sarama 2004; Sztajn et al. 2012). Both teachers and students need to be able to answer crucial questions:

- What mathematics is being learned?
- Why is it important?
- How does it relate to what has already been learned?
- Where are these mathematical ideas going?

Situating learning goals within the mathematical landscape supports opportunities to build explicit connections so that students see how ideas build on and relate to one another and come to view mathematics as a coherent and connected discipline (Fosnot and Jacob 2010; Ma 2010).

The mathematical purpose of a lesson should not be a mystery to students. Classrooms in which students understand the learning expectations for their work perform at higher levels than classrooms where the expectations are unclear (Haystead and Marzano 2009; Hattie 2009). Although daily goals need not be posted, it is important that students understand the mathematical purpose of a lesson and how the activities contribute to and support their mathematics learning. Goals or essential questions motivate learning when students perceive the goals as challenging but attainable (Marzano 2003; McTighe and Wiggins 2013). Teachers can discuss student-friendly versions of the mathematics goals as appropriate during the lesson so that students see value in and understand the purpose of their work (Black and Wiliam 1998a; Marzano 2009). When teachers refer to the goals during instruction, students become more focused and better able to perform self-assessment and monitor their own learning (Clarke, Timperley, and Hattie 2004; Zimmerman 2001).

A clear grasp of the mathematics frames the decisions that teachers make as they plan mathematics lessons, make adjustments during instruction, and reflect after instruction on the progress that students are making toward the goals. In particular, by establishing specific goals and considering how they connect with the broader mathematical landscape, teachers are better prepared to use the goals to make decisions during instruction (Hiebert et al. 2007). This includes facilitating meaningful discourse, ensuring connections among mathematical ideas, supporting students as they struggle, and determining what counts as evidence of students' learning (Seidle, Rimmele, and Prenzel, 2005). The practice of establishing clear goals that indicate what mathematics students are learning provides the starting point and foundation for intentional and effective teaching.

Illustration

Establishing clear goals begins with clarifying and understanding the mathematical expectations for student learning. Figure 2 presents an excerpt from a session in which two teachers, Ms. Burke and Mr. Miller, together with their math coach, engage in a collaborative planning session to discuss and clarify the mathematics learning goals for their second-grade students. Notice how the teachers begin by describing what the students will be doing in the lesson, rather than what they will be learning. Of course, teachers need to attend to the logistics of a lesson, but they must also give sufficient attention to establishing a detailed understanding of the mathematics learning goals. Consider how the math coach intentionally shifts the conversation to a discussion of the mathematical ideas and learning that will be the focus of instruction.

Two classes of second-grade students are currently working on understanding and solving addition and subtraction problems set in real-world situations. The following conversation develops among two teachers and their math coach in a planning session. The teachers have selected three story problems to give meaning to subtraction and serve as a focus for one of the lessons:

- Morgan wants to buy the next book in her favorite series when it is released next month. So far, she has saved $15. The book will cost $22. How much more money does Morgan need to save so that she can buy the book? (Problem type: Add to/Change Unknown)
- George and his dad are in charge of blowing up balloons for the party. The package had 36 balloons in it. After blowing up many balloons, George's dad noticed that the package still contained 9 balloons. How many balloons had they blown up? (Problem type: Take from/Change Unknown)
- Lou and Natalie are preparing to run a marathon. Lou ran 43 training miles this week. Natalie ran 27 miles. How much farther did Lou run than Natalie? (Problem type: Compare/Difference Unknown)

Ms. Burke: I think we should have the students work together in small groups to solve the word problems.

Mr. Miller: I agree, and they could take turns reading the problems, and then everyone could draw diagrams or use cubes to solve them, and then they could compare their answers.

Fig. 2. Collaborative planning session focused on clarifying mathematics goals for a lesson on problem situations for subtraction

Math Coach:	OK, that's what you want the students to do. So now let's talk more about what is it that you want your students to learn as a result of this lesson.
Ms. Burke:	We want them to better understand these different types of word problems and be able to solve them.
Math Coach:	OK. So, let's list some of the indicators that would show they understand.
Mr. Miller:	They would be able to use cubes or draw diagrams to show what is happening in the problem, explain what they did and why, and be able to get the right answer.
Ms. Burke:	I also want them to write an equation that models each situation. Some of the equations might be 15 + ☐ = 22, 36 = ☐ + 9 or 36 – ☐ = 9, and 43 – 27 = ☐ or 43 = 27 + ☐.
Mr. Miller:	Then if we have time in this lesson, or maybe the next day, we want the students to compare the different problems and equations and be able to explain how these relate to addition and subtraction, even though the contexts seem so different.
Math Coach:	Can you say a little more about why you picked these three problems for this lesson?
Mr. Miller:	Each word problem is about a different situation that gives meaning to subtraction. One problem is about finding an unknown addend, one is about subtraction as taking away, and the other is about finding the difference when comparing two amounts.
Ms. Burke:	We are hoping that the students get better at thinking about the relationships among the quantities in each context and how this relates to addition and subtraction. And they need to be able to work with these harder problem types and not just the easy take-away word problems [*i.e., Take from/Result Unknown*].
Math Coach:	Let me see if I can summarize this for us. Your learning goals for these lessons are for the students to represent and solve word problems by using diagrams or objects and equations, compare how the problem situations are similar and different, and explain how the underlying structure in each problem relates to addition and subtraction.
Ms. Burke:	Yes, and in their explanations, I want to hear them talk about what each number means in the problem, so in this lesson they know the total amount and one of the parts or addends, and they need to find the other unknown addend.

Note: Classification of problem types is based on CCSSM Glossary, Table 1 (NGA Center and CCSSO 2010, p. 88).

Fig. 2. *Continued*

As a result of the planning conversation, the teachers have a more precise understanding of the addition and subtraction concepts that they hope will surface during the lesson. For example, they expect their students to connect math drawings and equations and compare the mathematical structures of the various types of problem situations. At the beginning of the lesson, they discuss with students the goal and importance of understanding different kinds of word problems by using math drawings and writing equations. During instruction, the teachers are attentive to ensuring that students are not just finding the answers to the word problems but are able to explain how each problem relates to addition and subtraction and how that relationship is reflected in their drawings and equations. This in turn will compel students to focus on the how these problem situations relate to addition and subtraction and why that is an important aspect in their learning of mathematics.

Teacher and student actions

Effective teaching requires a clear understanding of what students need to accomplish mathematically. Clear learning goals focus the work of teaching and student learning. Teachers need to establish clear and detailed goals that indicate what mathematics students are learning, and they need to use these goals to guide decision making during instruction. Students also need to understand the mathematical purpose of a lesson. Teachers should help students understand how specific activities contribute to and support the students' learning of mathematics as appropriate during instruction. Students can then gauge and monitor their own learning progress. The actions listed in the table below provide guidance on what teachers and students do in establishing and using goals to focus learning in the mathematics classroom.

Establish mathematics goals to focus learning
Teacher and student actions

What are *teachers* doing?	What are *students* doing?
Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit.	Engaging in discussions of the mathematical purpose and goals related to their current work in the mathematics classroom (e.g., What are we learning? Why are we learning it?)
Identifying how the goals fit within a mathematics learning progression.	Using the learning goals to stay focused on their progress in improving their understanding of mathematics content and proficiency in using mathematical practices.
Discussing and referring to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning.	Connecting their current work with the mathematics that they studied previously and seeing where the mathematics is going.
Using the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction.	Assessing and monitoring their own understanding and progress toward the mathematics learning goals.